

# Example on Hydrostatic forces on submerged plane surfaces

- Example :
- The rigid gate,  $OAB$  is hinged at  $O$  and rests against a rigid support at  $B$ . What minimum horizontal force,  $P$ , is required to hold the gate closed if its width is 3 m? Neglect the weight of the gate and friction in the hinge. The back of the gate is exposed to the atmosphere.

$$F_1 = \gamma h_{c_1} A_1 \quad \text{where } h_{c_1} = 5\text{m}$$

Thus,

$$F_1 = \left(9800 \frac{\text{N}}{\text{m}^3}\right) (5\text{m}) (4\text{m} \times 3\text{m})$$

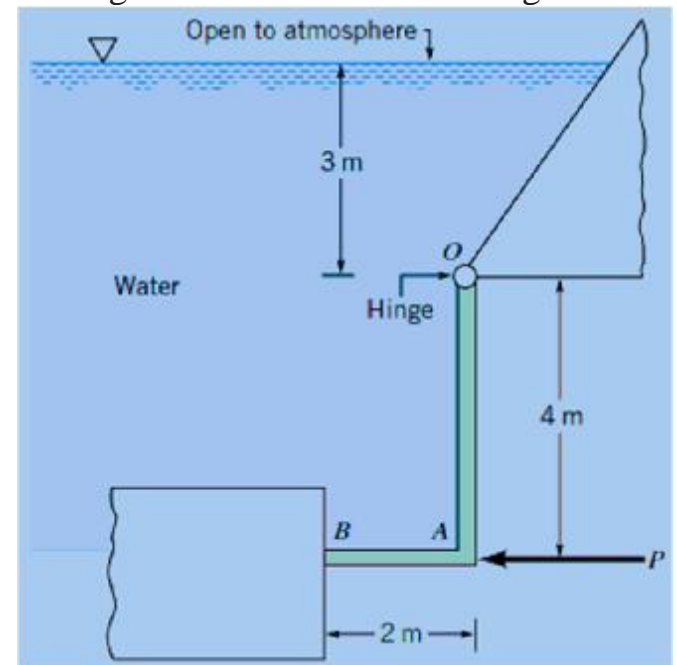
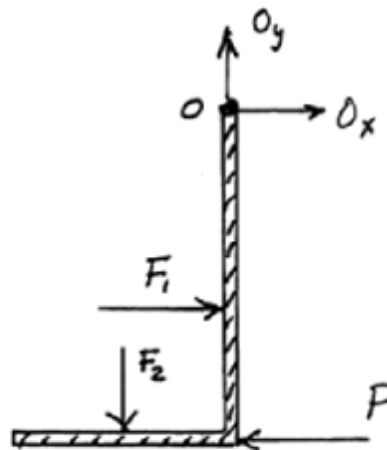
$$= 5.88 \times 10^5 \text{ N}$$

$$F_2 = \gamma h_{c_2} A_2 \quad \text{where } h_{c_2} = 7\text{m}$$

so that

$$F_2 = \left(9800 \frac{\text{N}}{\text{m}^3}\right) (7\text{m}) (2\text{m} \times 3\text{m})$$

$$= 4.12 \times 10^5 \text{ N}$$



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- Example :

To locate  $F_1$ ,

$$y_{R_1} = \frac{I_{xc}}{y_c A_1} + y_{c_1} = \frac{\frac{1}{12} (3\text{m})(4\text{m})^3}{(5\text{m})(4\text{m} \times 3\text{m})} + 5\text{m} = 5.267\text{m}$$

The force  $F_2$  acts at the center of the AB section. Thus,

$$\sum M_O = 0$$

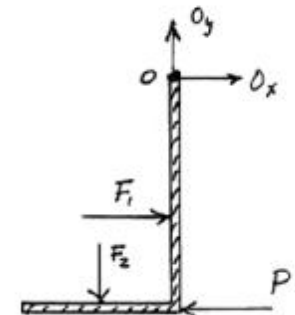
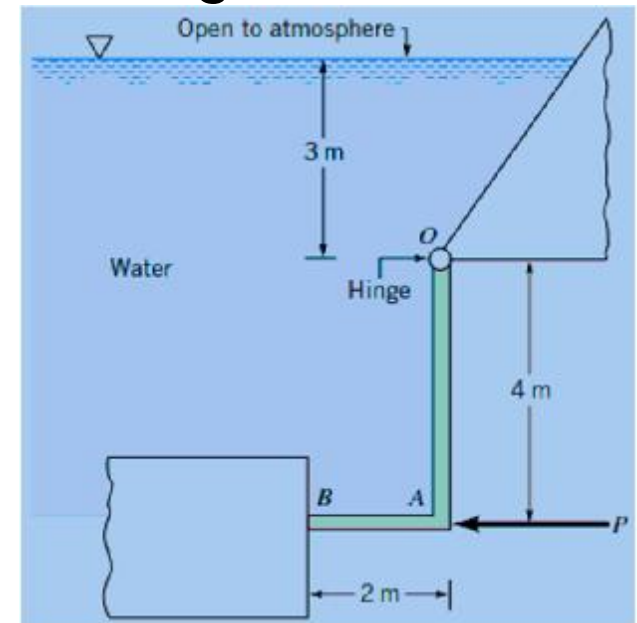
and

$$F_1 (5.267\text{m} - 3\text{m}) + F_2 (1\text{m}) = P (4\text{m})$$

so that

$$P = \frac{(5.88 \times 10^5 \text{ N})(2.267\text{m}) + (4.12 \times 10^5 \text{ N})(1\text{m})}{4\text{m}}$$

$$= \underline{\underline{436 \text{ kN}}}$$



# Hydrostatic forces on submerged curved surfaces

- In order to determine  $F_R$  on two-dimensional curved surface, you have to determine the horizontal and vertical components  $F_H$  and  $F_V$  separately.
- Based on **Newton's third law**, the resultant force acting on a curved solid surface is equal and opposite to the force acting on the curved liquid surface.

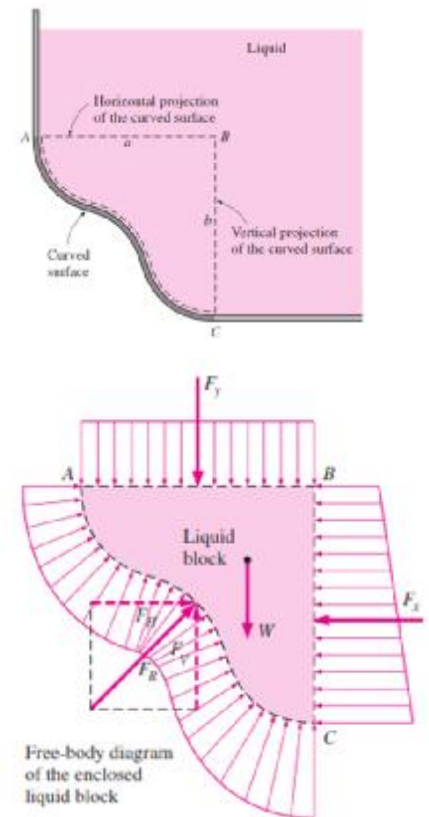
Balance of horizontal forces  $F_H = F_x$

Balance of vertical forces  $F_V = F_y + W$

Resultant force  $F_R = \sqrt{F_H^2 + F_V^2}$

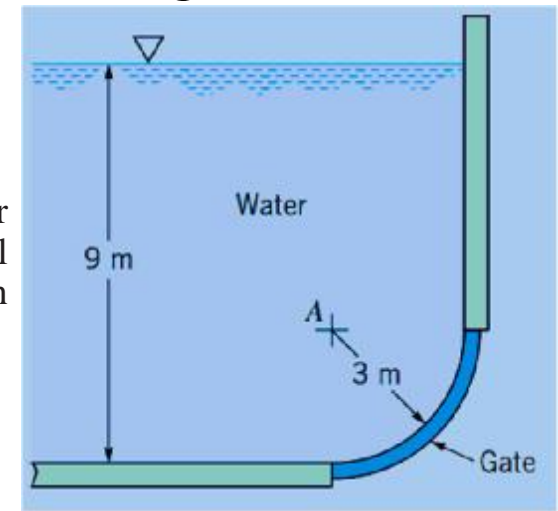
Angle between line of action of  $F_R$  and horizontal can be determined by

$$\tan \alpha = \frac{F_V}{F_H}$$



# Example on Hydrostatic forces on submerged curved surfaces

- Example :
- A 4-m-long curved gate is located in the side of a reservoir containing water as shown in the Fig. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate. Will this force pass through point A? Explain.



For equilibrium,

$$\sum F_x = 0$$

or

$$F_H = F_2 = \gamma h_{c2} A_2 = \gamma (6\text{ m} + 1.5\text{ m})(3\text{ m} \times 4\text{ m})$$

so that

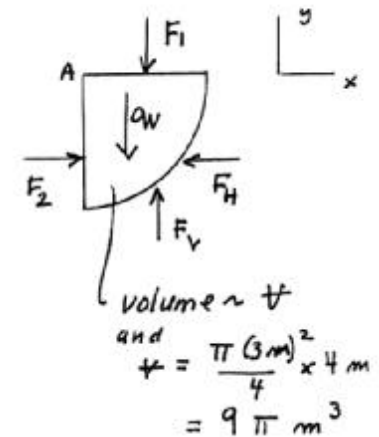
$$F_H = \left( 9.80 \frac{\text{kN}}{\text{m}^3} \right) (7.5\text{ m}) (12\text{ m}^2) = \underline{\underline{882\text{ kN}}}$$

Similarly,

$$\sum F_y = 0$$

$$F_V = F_1 + QW \quad \text{where :}$$

$$F_1 = [\gamma (6\text{ m})] (3\text{ m} \times 4\text{ m}) = \left( 9.80 \frac{\text{kN}}{\text{m}^3} \right) (6\text{ m}) (12\text{ m}^2)$$



# Example on Hydrostatic forces on submerged curved surfaces

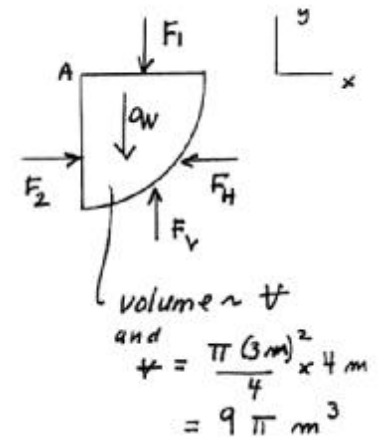
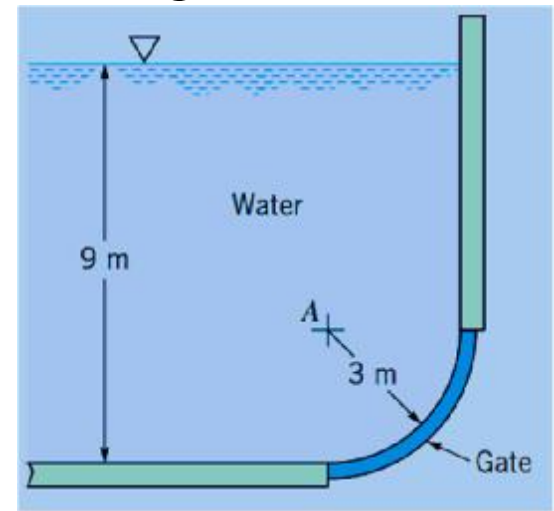
- Example :

$$Q_w = \gamma V = \left(9.80 \frac{\text{kN}}{\text{m}^3}\right) (9\pi \text{ m}^3)$$

$$\text{Thus, } F_V = \left(9.80 \frac{\text{kN}}{\text{m}^3}\right) [72 \text{ m}^3 + 9\pi \text{ m}^3] = \underline{\underline{983 \text{ kN}}}$$

(Note: Force of water on gate will be opposite in direction to that shown on figure.)

The direction of all differential forces acting on the curved surface is perpendicular to surface, and therefore, the resultant must pass through the intersection of all these forces which is at point A. Yes.



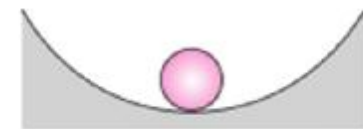
# Stability of immersed and floating bodies

- Stability of instability concepts

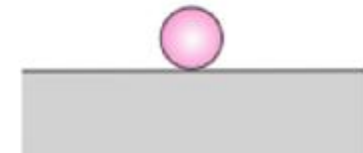
Case (a) is **stable** since any small disturbance (someone moves the ball to the right or left) generates a restoring force (due to gravity) that returns it to its initial position.

Case (b) is **neutrally stable** because if someone moves the ball to the right or left, it will stay put at its new location. It has no tendency to move back to its original location, nor does it continue to move away.

Case (c) is a situation in which the ball may be at rest at the moment, but any disturbance, even an infinitesimal one, causes the ball to roll off the hill—it does not return to its original position; rather it **diverges** from it. This situation is **unstable**.



(a) Stable



(b) Neutrally stable

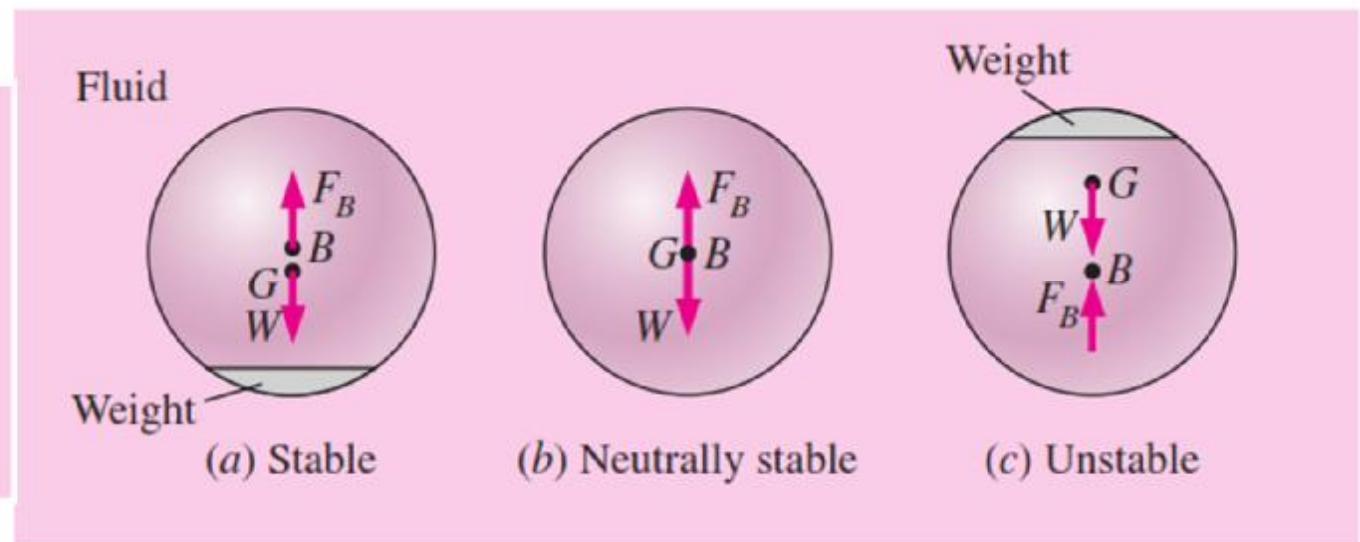
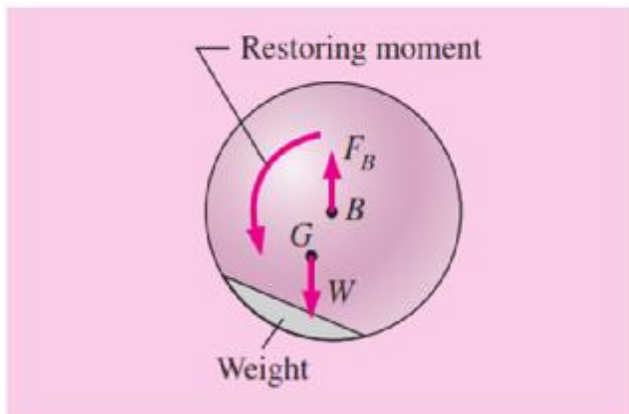


(c) Unstable

# Stability of immersed and floating bodies

- The *rotational stability of an immersed body*

depends on the relative locations of the *center of gravity*  $G$  of the body and the *center of buoyancy*  $B$ , which is the centroid of the displaced volume.

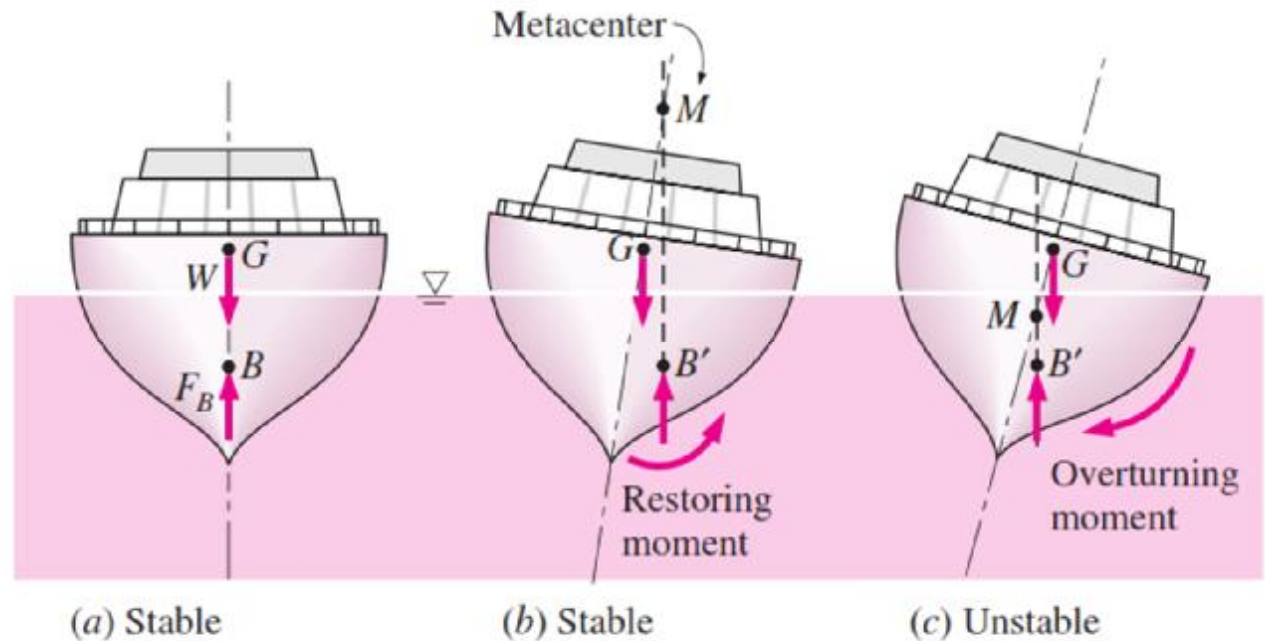


# Stability of immersed and floating bodies

- Stability of floating bodies

- The measure of stability for floating bodies is the *metacentric height GM*.

*Point M*: the intersection point of the lines of action of the buoyant force through the body before and after rotation.

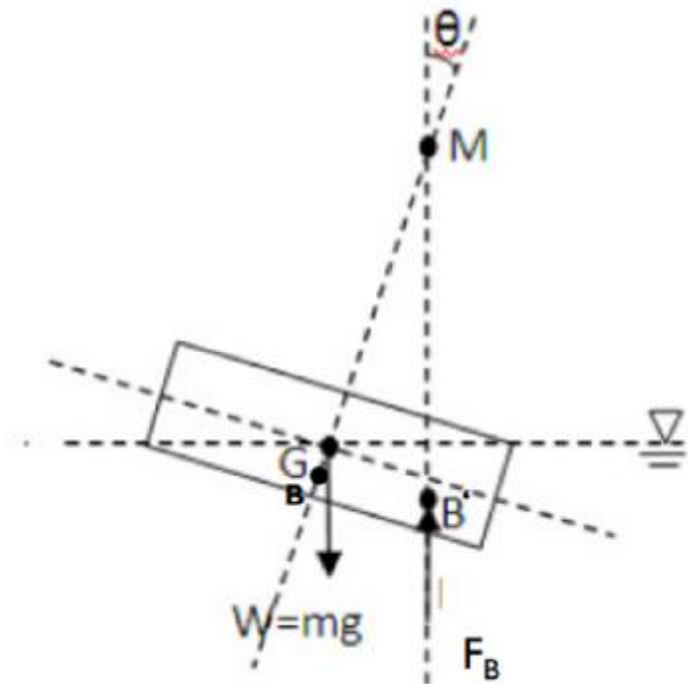




# Stability of immersed and floating bodies

- ▶ **Metacentric height (GM):** The distance between the center of gravity (G) of floating body and the metacenter (M) is called metacentric height. (i.e., distance GM shown in fig)

$$GM = BM - BG$$



# Stability of immersed and floating bodies

- Example :
- A solid cylinder 2 m in diameter and 2 m high is floating in water with its axis vertical. If the specific gravity of the material of cylinder is 0.65 find its metacentric height. State also whether the equilibrium is stable or unstable.

Size of solid cylinder= 2m dia, & 2m height

Specific gravity solid cylinder=0.65

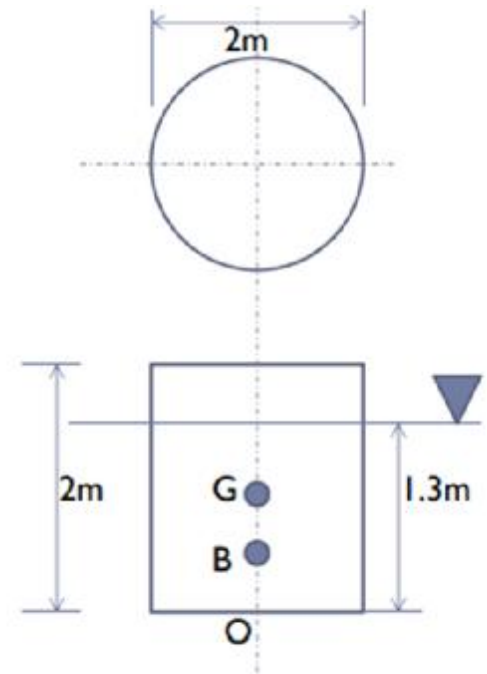
Let  $h$  is depth of immersion=?

For equilibrium

Weight of water displaced = weight of wooden block

$$9.81(\pi/4(2)^2(h))=9.81(0.65).(\pi/4(2)^2(2))$$

$$h=0.65(2)=1.3\text{m}$$



# Stability of immersed and floating bodies

- Example :

Center of buoyancy from O =  $OB = 1.3/2 = 0.65\text{m}$

Center of gravity from O =  $OG = 2/2 = 1\text{m}$

$BG = 1 - 0.65 = 0.35\text{m}$

Also;  $BM = I/V$

Moment of inertia =  $I = (\pi/64)(2)^4 = 0.785\text{m}^4$

Volume displaced =  $V = (\pi/4)(2)^2(1.3) = 4.084\text{m}^3$

$BM = I/V = 0.192\text{m}$

$GM = BM - BG = 0.192 - 0.35 = -0.158\text{m}$

-ve sign indicate that the metacenter (M) is below the center of gravity (G), therefore, the cylinder is in **unstable equilibrium**

